A-infinity GL(N)-equivariant matrix integrals-I

Serguei Barannikov

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The noncommutative Batalin-Vilkovisky equation and A-infinity algebras

- U -$\mathbb{Z}/2\mathbb{Z}$ graded vector space/$\mathbb{C}$, I-scalar product on U of degree $d \in \mathbb{Z}/2\mathbb{Z}$, (variant: $\mathbb{Z}-$graded), consider

$$F = Symm(C_\lambda[1+d]), \ C_\lambda = \oplus_{j=0}^{\infty}(U[1]^{\otimes j})^{\mathbb{Z}/j\mathbb{Z}}$$

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- $A_\infty-$algebras without scalar product are included in the formalism by setting $U = A \oplus A^\vee[d]$, giving an $A_\infty-$algebra with scalar product.

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$$\int \exp \widehat{S}(X, \Lambda) \widehat{\varphi} dX, \ \ \varphi \in Ker(\hbar\Delta + \{S, \cdot\}) \subset Symm(C_\lambda[1+d])$$

Noncommutative Batalin-Vilkovisky operator ([B1])

- I define my noncommutative BV differential on $Symm(C_\lambda[1+d])$ via

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- signs are the standard Koszul signs taking into account that $\overline{(x_{\rho_1}\dots x_{\rho_r})_\lambda} = (1+d) + \sum\overline{x_{\rho_i}}$, $x_i \in V[1]$.

- Invariant theory:

$$Symm(C_\lambda) \to Symm((gl(N|\widetilde{N}) \otimes V[1])^\vee)^{GL(N|\widetilde{N})}$$
$$\prod (a_1 \ldots a_k)_\lambda \to \prod sTr(A_1 \cdot \ldots \cdot A_k)$$

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- Such algebra exists:

$$q(N) = \{[X, \pi] = 0 | X \in gl(N|N)\}$$

where $\pi = \begin{pmatrix} 0 & 1_N \\ -1_N & 0 \end{pmatrix}$ is an odd isomorphism, $\pi^2 = -1$,
$q(N) = \left\{ X = \begin{pmatrix} A & B \\ B & A \end{pmatrix} \right\}$

Algebra q(N) and nc-BV equation

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- Corollary: tensor multiplication by $q(N)$, $gl(N|\widetilde{N})$ $\to$super Morita equivalence on solutions to nc-BV.

- $S \in Symm(\Pi C_\lambda) \to \widehat{S} \in Symm((q(N) \otimes V[1])^\vee)^{Q(N)}$, superfunction on affine BV manifold

Integration of polyvectors - the (equivariant) BV-formalism

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- action by the *super*-Lie algebra $[\Lambda, \cdot] \to$extension to $gl-$equivariantly closed differential form.

New integration framework in the noncommutative (derived algebraic) geometry,

- A.Connes: integration theory based on maps $C_\lambda \to$differential forms-"cycles" on $HC^*(A)$, $(\Omega, d, \int)$,

$$\tau(a_0, \ldots, a_n) = \int a_0 da_1 \ldots da_n$$

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- Invariance with respect to $A_\infty-$gauge transformation (and more general gauge transformation)

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- action by $[\Lambda, \cdot]$, $\Lambda \in q_{odd}$, $\rightarrow$ extension of $\exp \frac{1}{3!} otr(\Xi^3) dX$ to equivariantly closed differential form $\exp \frac{1}{3!} otr(\Xi^3) + \frac{1}{2} otr([\Lambda, \Xi], \Xi) dX$

Example: V={e} and matrix Airy integral

- $V = \{e\}$, $e \cdot e = e$, $I^\vee(e) = 1$, $\rightarrow$potential $(\xi, \xi, \xi)_\lambda$
- on $(q \otimes \Pi V)_0$ this gives the nonhomogenious polyvector $otr(\Xi^3)$,
- action by $[\Lambda, \cdot]$, $\Lambda \in q_{odd}$, $\rightarrow$ extension of $\exp \frac{1}{3!} otr(\Xi^3) dX$ to equivariantly closed differential form $\exp \frac{1}{3!} otr(\Xi^3) + \frac{1}{2} otr([\Lambda, \Xi], \Xi) dX$
- its highest degree component is the matrix Airy integral

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